

# A Dynamical Crossover Regime in the Transmission and Reflection Spectra of Evanescent Waves with 2D Arrays of Josephson Junctions

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## Abstract

A dynamical cross-over regime is revealed when exposing a classical two-dimensional ordered Josephson junction (JJ) array to evanescent waves and tuning the incident microwave power. At the lowest possible temperature for these experiments, 1.1 K, and at the lowest power setting, -55 dBm, evanescent waves are transmitted without loss and the resonance exhibits a quality factor of  $\sim 4200$ . A second, smaller resonance, which evolves with increasing power from the main resonance, is also investigated. In contrast to the behavior of the main resonance, this second peak grows as the incident power is increased and does not maintain a fixed resonant frequency for temperatures less than the superconducting critical temperature of niobium. The tunability of both resonances is studied as a function of temperature and microwave power. Finally we speculate that this dynamical crossover regime is evidence of a transition between two states of phase coherence where at low microwave power the JJ arrays are phase locked and at high microwave power the JJ arrays are unlocked.

Negative index of refraction metamaterials, materials that simultaneously have a negative permittivity and negative permeability, bend electromagnetic waves in counterintuitive directions.[1] One promise of these materials is their ability to transform decaying evanescent waves into amplifying waves and, by doing so, extend the resolution of optical microscopes to sub-wavelength range. [2] However, this opportunity [3] is jeopardized by dissipative losses which grow as the size of the constituents, the meta-atoms, are made smaller.[4] An encouraging remedy to overcome losses is to directly embed gain media into metamaterials having fishnet geometries. Theoretical models suggest that as a result of loss compensation, electromagnetic signals can be amplified with 'high' quality factors, as 'high' as 800, for realistic values of gain.[5] While amplifying *propagating* electromagnetic waves has successfully been demonstrated, albeit with signals having a low quality factor,  $Q < 50$ , [6] a similar, tunable amplification of *non-propagating* evanescent waves with high quality factors remains to be seen. And since losses and lack of tunability are the real show stoppers for future applications, achieving the remarkable properties known to metamaterials demands removing nearly all losses [7] while embedding nonlinear elements for tunability.

Lossless materials which combine fishnet, or square loop, geometries with superconducting nonlinear gain media are known as two dimensional ordered Josephson junction arrays. Josephson junction (JJ) arrays have intrinsic capacitances and inductances built into them - a key component in the design of negative index of refraction metamaterials. As attractive tunable metamaterials their salient features are their low loss nature, nonlinear inductance, and miniaturized device elements.[8] All of this leads to materials that are well suited as metamaterials but in practicality, JJ arrays, at least conventional JJ arrays, would not be of convenience since they need

to be kept cold and radiated with microwaves. Nevertheless, if this is done, then these are ideal materials for restoring decaying evanescent waves.

In this Letter, we report low temperature measurements of evanescent wave amplification with quality factors as high as 4200 when using unbiased superconducting JJ arrays. Evanescent wave amplification, as described in the transmission experiments of Ref [9], is the sharp increase in transmission through a waveguide with a metamaterial in the same frequency range as the cutoff regime of an empty waveguide. We also observe tunability and a surprising transition between two different sets of resonances with drastically different responses to temperature and incident power. Once the temperature exceeds the superconducting transition temperature, the two tunable resonances no longer prevail suggesting their relevance to the sample's superconducting properties.

The samples are two dimensional JJ arrays that were commercially fabricated by the Sperry-Univac Corporation, now Unisys, using the selective niobium anodization process [10] and constructed on a silicon substrate covered with a  $1 \mu\text{m}$  thick oxide layer. [11] These arrays, arranged in a square ring geometry are unique in that there is no ground plane. Each ring has four equally spaced Josephson junctions and is defined here as a "meta-atom" whose size is much smaller than the applied microwave wavelength, thus creating an effective medium. The length of one side is 29 microns, with a linewidth of 5 microns and a niobium thickness of 500 nm. The junctions have an area of  $(2.5 \text{ microns})^2$  and are made of Nb-amorphous Si-Nb, with a-Si serving as a 10 nm thick insulating barrier. A portion of the JJ arrays is shown in Fig.1a. The critical current of an individual junction is estimated to be  $1.65 \times 10^{-7} \text{ A}$ . Transport measurements in a magnetic field on these arrays are reported elsewhere. [12] The superconducting transition temperature is  $T_c = 9.2 \text{ K}$  as measured by

SQUID magnetometry.

The original sample with one million ( $1000 \times 1000$ ) junctions is divided into several pieces with each piece containing only the arrays and *not* electrodes or pads for electrical contacts. The samples are placed in a Ag-plated Cu X-band rectangular waveguide and held in place with a piece of Rohacell, a styrofoam-like material with a dielectric constant between 1.06 - 1.11 and oriented such that the rf magnetic field is perpendicular to the surface of the sample.

Attached to the rectangular waveguide are heaters, a thermometer and two antennas which are housed in a vacuum can inside a magnetically shielded cryostat. Semi-rigid coaxial cables [13] connect to the antennas and extend to a HP 8722 D vector network analyzer via phase-maintaining coaxial cables. [14] Standard transmission and reflection measurements are made by acquiring scattering parameters; the data is averaged over 16 subsequent runs with 1601 data points taken for each run. No additional filters or amplifiers are added to the system. The cutoff frequency for an empty waveguide is 6.56 GHz as shown in Fig. 1b.

We place several small, unbiased samples of different sizes in the waveguide. At room temperature, well above  $T_c$  of the JJ arrays, the electromagnetic transmission has broad ripples and a broad resonance at 12 GHz as shown in the blue curve of Fig. 1c. Additionally, there is noticeable insertion loss of -7 dB and the signal from 5 GHz to the cutoff frequency is much broader than for the empty waveguide. By cooling to liquid helium temperatures, where the arrays are superconducting, several sharp resonances appear at discrete frequencies both above and below cutoff as shown in the red curve of Fig. 1c. Also, the insertion loss goes to zero indicating that the lossy spectrum at room temperature is a consequence of the normal state JJ arrays and not the substrate.

To explore these resonant features in greater detail and without the possibility of crosstalk, we place a single sample of 0.5 cm x 0.5 cm dimensions with roughly 30,000 meta-atoms at the center of the waveguide. At fixed temperature,  $T = 1.1$  K, two distinctive transmission features appear below cutoff with increasing incident power from -55 dBm to -5 dBm having drastically different behaviors: the dominant resonant peak *decreases* in amplitude at fixed frequency then slightly shifts to lower frequencies over a range of  $\sim 0.4$  MHz, until a second, smaller resonance develops and *increases* in amplitude with increasing frequency over a range of  $\sim 13$  MHz as shown in Fig. 2a. Increasing the power also has the effect of broadening the dominant resonance, Q decreases from 4200 to 340, while sharpening the smaller resonance, Q increases from 340 to 1240. The amplitude of the dominant resonance is tunable over 30 dBm with a "net gain" of 20 dB, and the smaller resonance is tunable over 10 dBm with a "net gain" of 8 dB. Here "net gain" refers to the increase in the signal's amplitude from negative dB to 0

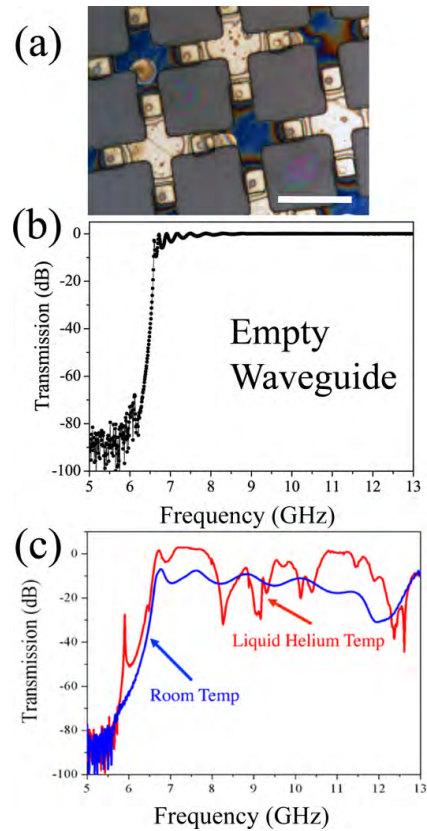


FIG. 1: (color online) (a) Optical microscope image of a portion of the sample. Each square loop contains 4 Josephson junctions; the scale bar denotes 30  $\mu\text{m}$ . (b) Plot of transmission vs frequency for an empty waveguide. (c) Plot of transmission vs frequency of the same waveguide as above but containing several unbiased samples at room (blue curve) and liquid helium (red curve) temperatures. Note: The value of +3 dB between 7-8 GHz for the liquid helium curve corresponds to 0 dB; this offset is due to the calibration being done at room temperature while the measurements are made at liquid helium temperatures.

dB; this is *not a real gain* since the signal never exceeds 0 dB.

The observed transmission of evanescent waves from one end of the waveguide to the other occurs by tunneling since evanescent waves cannot carry energy. The signature of 'transmission by tunneling' is equal, or near equal, amplitudes of forward and backward evanescent modes [9],[15]; this is shown in the reflection and transmission coefficients and plotted as a function of frequency for -55 dBm and -5 dBm in Fig. 2b.[16]

The resonant frequency,  $f$ , of a single JJ meta-atom is estimated from the geometric inductance,  $L_g$ , the Josephson inductance,  $L_J$ , and the capacitance,  $C$ , of the tunnel junctions. This is expressed as  $f = \frac{1}{2\pi} \frac{1}{\sqrt{\left(\frac{4L_J L_g}{4L_J + L_g}\right) \times 4C}}$  with  $L_g = \mu_0 R \left[ \ln\left(\frac{8R}{r}\right) - \frac{7}{4} \right]$  where  $R$  is the radius of the square loop and  $r$  is the radius of the wire. [17] From the

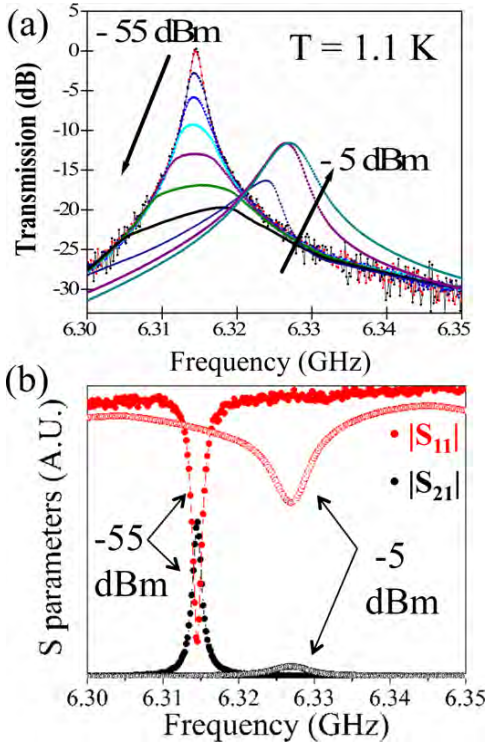


FIG. 2: (color online) Measurements at  $T = 1.1$  K. (a) Frequency dependence of array transmission with input power ranging from -55 dBm to -5 dBm, every -5 dBm. The arrows indicate increasing input power. (b) Magnitude of the transmission  $|S_{21}|$  (black circles) and reflection  $|S_{11}|$  (red circles) at -55 dBm and -5 dBm. [16]

dimensions of an JJ meta-atom and the Josephson inductance which is defined as  $L_J = \frac{\Phi_0}{2\pi I_c}$  with  $\Phi_0 = \frac{h}{2e}$  where  $I_c$  is the critical current,  $h$  is Planck's constant, and  $e$  is the charge of the electron, the estimated resonant frequency is  $\sim 50$  GHz. This crude estimate is an order of magnitude higher than the observed resonant frequency suggesting that additional terms have been neglected, such as the kinetic inductance or possibly the estimated critical current is incorrect. In addition, this calculation is not accurate in describing the experimental situation since the resonant frequency depends on the location and orientation of the sample inside the waveguide. We compare this calculation to the Josephson plasma frequency,

$$f = \frac{1}{2\pi} \sqrt{\frac{2eI_c}{\hbar C}} \sim 15 \text{ GHz}, \text{ which is closer to the actual value.}$$

Given the discrepancies between predicted values and experimental observations, it is necessary to relate the observations to superconductivity and the Josephson effect. For superconductivity, microwave measurements at different temperatures are made, with four different temperatures shown in color plots of Fig. 3. Perfect transmission, 0 dB, at  $T = 1.1$  K and 6.0 K for -55 dBm is shown in green in Figure 3 a and b.

Interestingly, as the temperature approaches  $T = 6.0$  K, higher and higher input powers are required for the emergence of the second peak; this behavior is clearly seen when comparing the transmission spectrum in Fig. 3b to that in Fig. 3a. One possible explanation for this is that with increasing temperature, until  $T = 6.0$  K, more junctions become coherent with each other as larger numbers of their individual linewidths overlap. Thermally induced coherence can be a result of the sample not being precisely centered in the waveguide or perfectly oriented with the rf magnetic field. The onset of coherence with temperature was previously mentioned in Ref. [18].

This situation drastically changes for  $T > 6.0$  K: the tunability of each resonance substantially decreases with increasing power. Indeed, since these are two dimensional JJ arrays, they are model systems for the Berezinskii-Kosterlitz-Thouless (BKT) phase transition. As previously reported for these samples, the BKT transition temperature between the binding of vortex and anti-vortex pairs ( $T < T_{BKT}$ ), and the unbinding of these pairs ( $T > T_{BKT}$ ) occurs at  $T_{BKT} = 5.85$  K. [12] Our data of a turning point at  $T = 6.0$  K is consistent with this picture. At  $T = 8.0$  K, the transmission of the smaller, higher frequency resonance saturates and is no longer tunable with increasing power as shown in Fig. 3c. However, the main resonance is still somewhat tunable at  $T = 8.0$  K, although transmission occurs at lower frequencies than the resonances at  $T \leq 6.0$  K. At  $T = 10$  K and beyond, there is only one broad, low amplitude resonance which is constant, albeit noisy, with incident power as shown in Fig. 3d; in aggregate, these temperature and incident power results are indicative of superconductivity.

Central to understanding the nonlinearity and Josephson effect of these spectra requires explaining the tunability of the 6.314 GHz resonance for  $T \leq 6.0$  K. If the reduction in amplitude with increasing power is a result of dissipation, then as  $T$  increases, the amplitude should decrease. Keeping power fixed at -55 dBm and increasing  $T$ , the amplitude decreases. However, the decrease in amplitude at fixed power with temperature is not of the same ilk as the decrease in amplitude at fixed  $T$  with increasing power as seen in Fig. 2a and Fig. 3. First, as  $T$  increases at fixed power, the resonant frequency shifts to lower values than for the case of increasing power at fixed  $T$  as shown in Fig. 4a. Secondly, and rather remarkably, tunability of the resonance occurs with very low incident power even though the niobium thickness is 500 nm. For even thinner films of niobium,  $\sim 200$  nm, without JJs, significantly higher values of incident power are necessary, as high  $\sim +20$  dBm, to lower the amplitude by  $\sim 20$  dB; for thin Nb films, this amplitude reduction, accompanied with a downward frequency shift, is due to dissipative losses. [19] Lastly, and most surprisingly, is the decrease in loss,  $1/Q$ , with increasing  $T$  for input powers between -40 dBm and  $\sim -18$  dBm for  $T < 6$  K. This is shown in the shaded region of Fig. 4b and confirms that dissipation

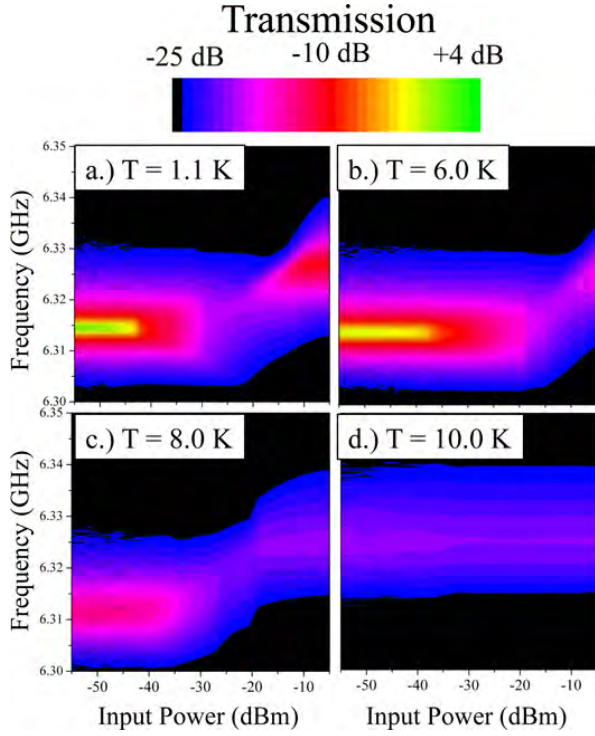


FIG. 3: (color online). Microwave transmission (dB) as a function of frequency and input power for a.)  $T=1.1$  K, b.)  $T=6.0$  K, c.)  $T=8.0$  K, and d.)  $T=10.0$  K. The color bar scale represents the range of transmission from zero transmission (black) to perfect transmission (green). Note: The value of +4 dB corresponds to 0 dB due to the calibration being done at room temperature while the measurements are made at liquid helium temperatures.

is not driving the decrease in amplitude with increasing input power of the main resonance.

If the tunability of the main resonance is not due to dissipation, then another possibility is that the JJs are operating as parametric amplifiers owing to their non-linear inductance. The similarity between these results and those due to parametric amplification might not be coincidental since the signal from the network analyser is not a pure tone, but rather contains sidebands. It is by these sidebands that we are likely seeing "gain". We measured the output signal at 6.4 GHz from the network analyzer; there are sidebands spaced at  $\pm 20$  kHz and  $\pm 50$  kHz as well as higher order harmonics. In addition to a central frequency, sidebands from separate sources are standard for amplifying signals with JJ arrays; however, in these experiments, the amplitude of the signal is always greater than 0 dB.[20] - [22]

The losses,  $1/Q$ , increase with increasing  $T$  for high power resonances, greater than -18 dBm, indicating increasing dissipation with temperature as shown in Figure 4 b. Additionally, since these resonances shift to higher frequencies with incident power as shown in Figs. 2 a

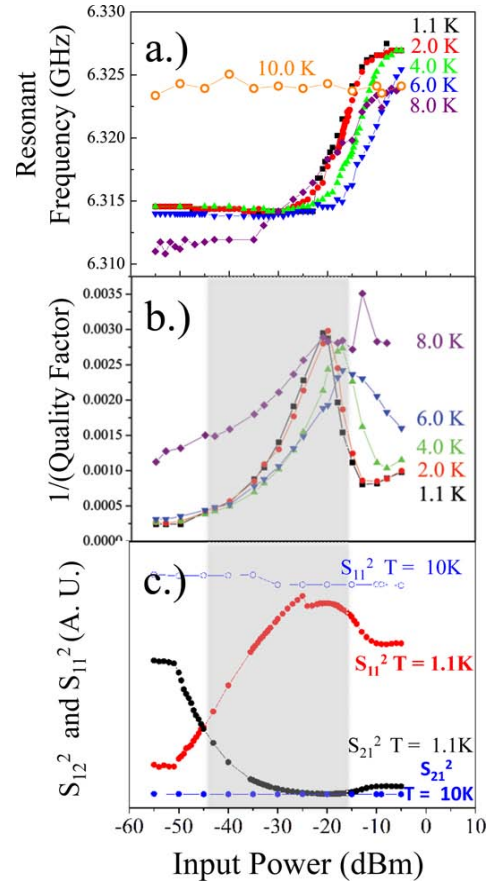


FIG. 4: (color online). (a) Resonant frequency vs input power for different temperatures. (b)  $(1/Q)$  vs input power for different temperatures. The shaded gray region, -40 dBm to -18 dBm, indicates a regime where  $1/Q$  decreases with increasing temperature from 1.1 K to 6.0 K. (c)  $S_{12}^2$  and  $S_{11}^2$  vs input power for  $T=1.1$  K and  $T=10$  K.

& 4 a this suggests that these resonances are not due to quasiparticles, but likely free vortices. When comparing  $1/Q$  to the transmission and reflection coefficients we observe that as  $1/Q$  increases or decreases, the reflection coefficients follow suit as shown for  $T=1.1$  K in the red curve in Fig. 4c. Above the superconducting transition temperature,  $T=10$  K, the signal is all reflection as shown in the open circles of the blue curve of Fig. 4c. So, why then, does one peak evolve into a second peak? Why are there two peaks? What is the nature of the second peak? These are left as open questions and require further investigation.

However, and without a corroborative theory to back up our findings, we naively speculate that the difference between these two sets of resonances is a measure of how phase sensitive the JJ arrays are to incident power: we speculate that the arrays are phase insensitive at low power and thus are phase locked for the dominant resonances, while for the second set of resonances, at higher



incident powers, the arrays are phase sensitive, and thus unlocked. This results in a constant resonant frequency for the phase locked arrays, but a shifting resonant frequency for the unlocked arrays with incident power.

We also base our thinking on eliminating what these resonances are not: first, we rule out generation of quasi-particles with increasing incident power for  $T \leq 6.0$  K because in our work increasing power shifts the resonances to higher frequencies not lower frequencies as would otherwise be the case. Second, we rule out extraordinary transmission [23] since the resonant frequency and sample location in the waveguide are coupled to each other.

In conclusion, tunable amplification of evanescent waves has been demonstrated using unbiased, fishnet Josephson junction metamaterials with a simple experimental setup that does not require additional filtering, amplifiers or biasing of the sample. This experimental simplicity together with the robustness and tunability of the evanescent signal over a wide range of incident powers and temperatures make these experimental results all that more compelling. Most noteworthy, however, is the unanticipated transition from one set of resonances to a second set of resonances. This transition could be a manifestation of the arrays transitioning from being locked in phase to being unlocked; however, further investigations are needed to confirm this speculation or provide an alternative explanation.

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